parallel of latitude ϕ' be zero; it is observed that, owing to the way μ has been found, the distortion on the parallel of ϕ'' will be zero likewise. Since

$$k = \frac{RdL}{r_1 d\lambda} = \frac{\mu R d\lambda}{r_1 d\lambda} = \mu \cdot \frac{R}{r_1},$$

we obtain

$$C = \frac{a \cos \phi'}{\mu \operatorname{tg}^{\mu} \zeta' \sqrt{1 - e^2 \sin^2 \phi'}}.$$

Since the cone is inside the earth's surface between the parallels ϕ' and ϕ'' and outside for the border area of the map, it follows that distances on the map will be smaller than corresponding distances on the earth's surface for the zone between ϕ' and ϕ'' and larger in the border-area. Computation will show that the maximum distortion amounts to 1 : 2037 or 5 : 10000; this, even for ranges of 10 km., will be a negligible quantity.

While geodetic lines on the surface of the earth are not represented by straight lines on the map but appear as slightly curved traces, the deviation for a distance of 100 km. will introduce an angular error in azimuth of less than one minute of arc.

In concluding this paper the reader's attention is once more called to Major Clark's able pamphlet which should prove a most interesting guide for students of surveying and cartography. Gretchel's *Lehrbuch der Karten-Projectionen* has been used in preparing this paper.

QUESTIONS AND DISCUSSIONS.

EDITED BY W. A. HURWITZ, Cornell University, Ithaca, N. Y.

REPLIES.

32. In a discussion of the Peaucellier[1] cell by analytic methods the following equations are obtained:

$$(1)\quad (x_2 - x_1)^2 + (y_2 - y_1)^2 - b^2 = 0; \qquad (2)\quad (x_3 - x_1)^2 + (y_3 - y_1)^2 - b^2 = 0;$$

$$(3)\quad (x_2 - X)^2 + (y_2 - Y)^2 - b^2 = 0; \qquad (4)\quad (x_3 - X)^2 + (y_3 - Y)^2 - b^2 = 0;$$

$$(5)\quad x_2^2 + y_2^2 - K^2 = 0; \qquad (6)\quad x_3^2 + y_3^2 - K^2 = 0;$$

$$(7)\quad x_1^2 + y_1^2 - 2cx_1 = 0.$$

The result of eliminating $x_1, y_1, x_2, y_2, x_3, y_3$ gives an equation of the first degree, which establishes that the linkage will trace a straight line. There are various ways of effecting this elimination.

1. What element of the situation is left unused by the following procedure in the elimination?
   (*a*) From equations (1), (3), (5) eliminate x_2 and y_2 and obtain an equation

$$(8)\quad f_1(x_1, y_1) = 0.$$

[1] In the accompanying figure, taken from the article on "Linkages" in the December, 1915, MONTHLY, by Mr. Leavens, the coördinates of the points of the linkage are: O (0, 0); C $(c, 0)$; P_1 (x_1, y_1); M (x_2, y_2); M' (x_3, y_3); P_2 (X, Y).

(b) From equations (2), (4), (6) eliminate x_3 and y_3 and obtain an equation

$$(9) \quad f_2(x_1, y_1) = 0.$$

(c) From equations (7), (8), (9) eliminate x_1 and y_1 and obtain the desired equation.

2. How should this procedure be supplemented to secure the result?

Reply by J. K. Whittemore, Yale University.

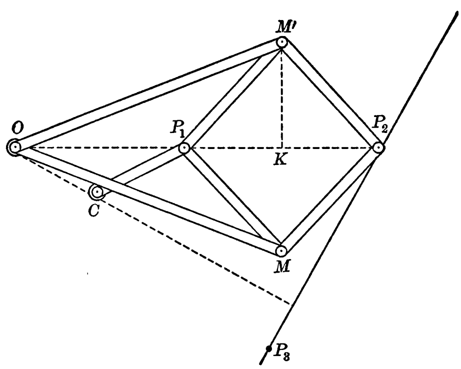
The method of elimination proposed might be expected to fail, since (8) and (9) are apparently identical, for equations (2), (4), (6) differ from (1), (3), (5) only in containing x_3, y_3 in place of x_2, y_2. But the functions f_1 and f_2 of (8) and (9) do not, as a matter of fact, exist at all. If we seek to determine f_1 we may write from (1) and (5)

$$2x_1x_2 + 2y_1y_2 = K^2 - b^2 + x_1^2 + y_1^2,$$

and from (3) and (5)

$$2Xx_2 + 2Yy_2 = K^2 - b^2 + X^2 + Y^2.$$

These equations may be formally solved for x_2, y_2 and the results substituted in (5), apparently giving f_1; but this work is illusory. Equations (2), (4), (6) give identically the same expressions for x_3, y_3, so that, unless (x_2, y_2) coincides with (x_3, y_3), the two equations above are equivalent and cannot be solved, and



$$(A) \quad \frac{x_1}{X} = \frac{y_1}{Y} = \frac{K^2 - b^2 + x_1^2 + y_1^2}{K^2 - b^2 + X^2 + Y^2}.$$

Since (x_2, y_2) and (x_3, y_3) do *not* coincide equations (A) are true. It is indeed obvious from the figure that $Yx_1 - Xy_1 = 0$. The equation of the locus of (X, Y) is obtained by eliminating x_1, y_1 from (A) and (7). The elimination is simply carried out as follows: Let the common value of the fractions in (A) be $\lambda \neq 1$. Then (7) gives

$$\lambda(X^2 + Y^2) - 2cX = 0.$$

Equating to λ the last member of (A),

$$(\lambda^2 - \lambda)(X^2 + Y^2) - (\lambda - 1)(K^2 - b^2) = 0.$$

Since $\lambda - 1 \neq 0$, this factor is cancelled from the last equation; then subtracting the preceding equation,

$$2cX = K^2 - b^2.$$

Note. This reply points out the necessity of using the fact that the points (x_2, y_2) and (x_3, y_3) are distinct and that the points (x_1, y_1) and (X, Y) are distinct, the latter condition being of course implied by $\lambda \neq 1$. It is likewise clear from the last part of the proof that neither (x_1, y_1) nor (X, Y) may coincide with $(0, 0)$; this may be insured by requiring that $K \neq b$. There are therefore several elements of the situation left unused by the process suggested in the question. —Editor.